Exercise 18

Solve the initial-value problem.

$$y'' - 2y' - 3y = 0, \quad y(0) = 2, \quad y'(0) = 2$$

Solution

This is a linear homogeneous ODE with constant coefficients, so its solutions are of the form $y = e^{rx}$.

$$y = e^{rx} \quad \rightarrow \quad \frac{dy}{dx} = re^{rx} \quad \rightarrow \quad \frac{d^2y}{dx^2} = r^2e^{rx}$$

Plug these formulas into the ODE.

$$r^2e^{rx} - 2(re^{rx}) - 3(e^{rx}) = 0$$

Divide both sides by e^{rx}.

$$r^2 - 2r - 3 = 0$$

Solve for r.

$$(r-3)(r+1) = 0$$

$$r = \{-1, 3\}$$

Two solutions to the ODE are e^{-x} and e^{3x}. By the principle of superposition, then,

$$y(x) = C_1e^{-x} + C_2e^{3x}.$$

Differentiate the general solution.

$$y'(x) = -C_1e^{-x} + 3C_2e^{3x}$$

Apply the initial conditions to determine C_1 and C_2.

$$y(0) = C_1 + C_2 = 2$$

$$y'(0) = -C_1 + 3C_2 = 2$$

Solving this system of equations yields $C_1 = 1$ and $C_2 = 1$. Therefore, the solution to the initial value problem is

$$y(x) = e^{-x} + e^{3x}.$$

Below is a graph of $y(x)$ versus x.